rectangled Cone whose Altitude is equal to *A R*. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is *A R*. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius *A R*.

The Description of the Rular and Wheel, *Fig.* 2. is sufficient for the Demonstration of the Properties of the Curve: but in order to an actual Construction for Use, I have added *Fig.* 5. where *A B* is a brass Rular; *w h* the little Wheel, which must be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. *s* represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixt Rular. *p* is a Stud for convenient holding the Rular in its Motion.

Note, *Most of these Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among those of the* Royal Academy *of* Sciences *for the Year* 1712. *but not publish'd till* 1715: *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL.D. & R. S. Secr. *By the Author.*

WHEN I apply'd my self to consider throughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir *Isaac Newton* our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Flux-

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ions of Quantities are proportional to the nascent Increments of those Quantities: in order to understand that Method throughly, I found it necessary to consider well the Properties of Increments in general. And from those Properties I saw it would be easy to draw a perfect Knowledge of the Method of Fluxions: for if in any case the Increments are supposed to vanish and to become equal to nothing, their Proportions become immediately the same with the Proportions of the Fluxions. In this Method I consider Quantities, as formed by a continual Addition of parts of a finite Magnitude, and those parts I call the Increments of the Quantities they belong to, because that by the Addition of them the Quantities are increased. These parts being consider'd as formed in the same manner by a continual Addition of other parts; thence follows the Consideration of second Increments; and so on to third, fourth, and other Increments of a higher kind. For Example, if x stands for any Number in the Series 0. 1. 4. 10. 20. 35, *&c.* in which the Numbers are formed by a continual Addition of the Numbers in the Series 1. 3. 6. 10. 15, *&c.* then the Numbers in the latter Series are call'd the Increments of the Numbers in the foregoing Series; thus, for Example, if to the third Number (4) in the first Series, I add the corresponding third Number (6) in the second Series, I shall produce the next, that is the fourth Number (10) in the first Series, and so the rest. Any Number in the first Series being call'd x, the corresponding Number (which is its Increment) in the second Series I express by $\dot{x}$. And these Numbers $\dot{x}$ being form'd in the same manner by the Numbers in the Series 1. 2. 3. 4, 5, *&c.* I call these last Numbers $\ddot{x}$, they being the first Increments of the Numbers $\dot{x}$, and the second Increments of the Numbers x; and so on. Hence having given any Series of Numbers that are call'd by a general Character x, their Increments are found by taking

their

their Differences; thus in the preſent Example, the firſt Increments $\underset{.}{x}$ in the Series 1. 3. 6. 10. 15, &c. are found by taking the Differences of the Numbers x in the Series 1. 4. 10. 20. 35, &c. and the ſecond Increments $\underset{..}{x}$ in the Series 1. 2. 3. 4. 5, &c. are found in the like manner, by taking the Differences of the Numbers $\underset{.}{x}$, and ſo of the third anj other Increments. This Method conſiſts of two parts; One is concerned in ſhewing how to find the Relations of the Increments of ſeveral variable Quantities, having given the Relations of the Quantities themſelves; and the Other is concerned in finding the Relations of the Integral Quantities themſelves freed from the conſideration of their Increments, having given the Relations of the Increments: either ſimply, or they being any how compounded with their Integral Quantities. In the Method of Fluxions Quantities are not conſider'd with their parts, but with the Velocities of the Motions they are ſuppoſed to be formed by; or to ſpeak more accurately, they are conſider'd with the Quantities of the Motions by which they are ſuppoſed to be generated; for the Fluxions are proportional to the Velocities, only when the moving Quantities, which produce the flowing Quantities conſider'd, are equal. Theſe Quantities of Motion, or Velocities when the moving Quantities are equal, are what Sir *Iſaac Newton* calls Fluxions. As in the Method of Increments there are ſecond, third, and other Increments; ſo in the Method of Fluxions there are ſecond, third, and other Fluxions; the Fluxions themſelves being conſider'd as Quantities that are formed by Motion, the Quantity of which Motion is their Fluxions. As the Method of Increments conſiſts of two Parts; one being concern'd in finding the Increments from the Integrals given, and the other in finding the Integrals having given the Increments; ſo does the Method of Fluxions conſiſt of two Parts; the one ſhewing how to find the Fluxions, having the

the Fluents given; and the other ſhewing how to find the Fluents freed from Fluxions, having given the Relations of the Fluxions, whether compounded with their Fluents or otherwiſe. The Principles of this Method may all be drawn directly as a Corollary from the Principles of the Method of Increments. For Sir *Iſaac Newton* having demonſtrated (*Phil. Nat. Princ. Math. Sect.* 1. and in the Beginning of his Treatiſe *De Quadratura Curvarum*) that the Fluxions of Quantities are proportional to their naſcent or evanaſcent Increments, if in any Propoſition relating to Increments, you make the Increments to vaniſh and to become equal to nothing, and for their Proportion put the Fluxions, you will have a Propoſition that will be true in the Method of Fluxions. This is but a Corollary to Sir *Iſaac Newton*'s Demonſtration of the Fluxions being proportional to the naſcent Increments. For this reaſon, to make the Method of Fluxions to be underſtood more throughly, I thought it proper to treat of theſe two Methods together, and I have handled them promiſcuouſly as if they were but one Method. Some people, becauſe that the Fluxions are proportional to the naſcent Increments of Quantities, have thought that by the Method of Fluxions Sir *Iſaac Newton* has introduced into Mathematicks rhe Conſideration of infinitely little Quantities; as if there were any ſuch thing as a real Quantity infinitely little. But in this they are miſtaken, for Sir *Iſaac* does only conſider the firſt or laſt Ratio's of Quantities, when they begin to be, or when they vaniſh, not after they are become ſomething, or juſt before they vaniſh; but in the very moment when they do ſo In this caſe Quantities are not conſider'd as infinitely little; but they are really nothing at the time that Sir *Iſaac* takes the Proportions of their Fluxions; and the Truth of this Method is demonſtrated from the Principles of the Method of Increments, in the ſame manner as the Ancients demonſtrated

ted their Conclusions in the Method of Exhaustions, by a *Deductio ad Absurdum.*

Having premised thus much in general concerning the two Methods here treated of, to come to a particular Description of this Book; In the Preface I give a short Description of the Method of Increments, and an Account of Sir *Isaac Newton*'s Notion of the Fluxions which I have already spoke of. The Book consists of two Parts, and contains 118 Pages in *Quarto*; the Propositions being number'd throughout from the Beginning. In the first Part I explain the Principles of both Methods: and in the second Part I shew the Usefulness of them in some particular Examples.

After having explain'd the Notation I make use of in the Introduction, in the first Proposition I explain the direct Method both of Increments and of Fluxions. The second Proposition shews how to transform an Equation wherein Integrals and their Increments, or wherein Fluents and their Fluxions are concern'd; so as in the Room of the Integrals or Fluents, to substitute their Compliments to a given Quantity with their Increments or their Fluxions, they increasing in a contrary Sense to the Quantities in the first Supposition. In the third Proposition I shew how to transform a fluxional Equation, so as to change the Characters of the Fluents, making that Quantity to flow uniformly which in the first Supposition flow'd unequally, having second, third and other Fluxions, and making that Quantity which in the first Supposition flow'd uniformly, now to flow unequally, so as to have second and third Fluxions, *&c.* This Proposition is of great use in the inverse Method, when you would invert the Expression of the Relation of the flowing Quantities; for Example, if in the Supposition z flows uniformly and x variably, by the inverse Method of Fluxions you will find x exprest by the Powers of z: but if you would find z

expreſt by the Powers of x, you muſt then transform the Equation by this Propoſition. Sir *Iſaac Newton* and Mr *de Moivre* do this by the Reverſion of Seriefes; but I take this to be the more proper and more genuine Method of doing it directly. In the fourth and fifth Propoſitions are explain'd the Method of judging of the Nature and Number of the Conditions that may accompany an Inctemental or a Fluxional Equation. This is a Circumſtance that I don't find to have been explain'd by any one before, and the Propoſitions are ſomething intricate; wherefore it will not be improper to explain this Matter a little more at large. Suppoſe then that z and x are two variable Quantities, and ſuppoſing z to increaſe uniformly by the continual Addition of its conſtant Increment $\underset{.}{z}$, (according to the Notation I make uſe of in this Book) ſuppoſe $x + z = x$. Then if it be propoſed as a Problem to find the Value of x, expreſs'd by the Powers of z, and quite freed from the Increments; by the fourth Propoſition there may be three Conditions added to this Problem. The Demonſtration of this is taken from the Formation of the Integrals by a continual Addition of their Increments. Suppoſe that all the Values of z, x, x, $\underset{..}{x}$, $\underset{\therefore}{x}$, *&c.* were ſet down in order in ſo many Columns, and that at the Head of the Table, the correſponding Values of $z, x, \underset{.}{x}, \underset{..}{x}, \underset{\therefore}{x}$, were expreſt by the Symbols a, c, $\underset{.}{c}$, $\underset{..}{c}$, $\underset{\therefore}{c}$. Then by means of the given Equation $\underset{\therefore}{x} + z = x$, or $\underset{\therefore}{x} = x - z$, will $\underset{\therefore}{c} = c - a$, whence the ſecond correſponding Values of z, x, $\underset{.}{x}$, $\underset{..}{x}$, $\underset{\therefore}{x}$ will be $a + \underset{.}{z}$, $c + \underset{.}{c}$, $\underset{.}{c} + \underset{..}{c}$, $\underset{..}{c} + c - a$ ($= \underset{..}{c} + \underset{\therefore}{c}$) and $c + \underset{.}{c} - a - \underset{.}{z}$ (by the Eq.) Whence again the third Values are $a + 2\underset{.}{z}$, $c + 2\underset{.}{c} + \underset{..}{c}$, $\underset{.}{c} + 2\underset{..}{c} + c - a$ ($= \underset{.}{c} + 2\underset{..}{c} + \underset{\therefore}{c}$) $\underset{..}{c} + 2c - 2a + \underset{.}{c} - \underset{.}{z}$, and $c + 2\underset{.}{c} + \underset{..}{c} - a - 2\underset{.}{z}$, and ſo you may proceed to find the fourth, fifth, and all the

the other Values of z, x, $\underset{\cdot}{x}$, $\underset{\cdot\cdot}{x}$, $\underset{\therefore}{x}$. But by what is already ſet down, it is evident that all their Values will be expreſt by a and $\underset{\cdot}{z}$, and the three Symbols c, $\underset{\cdot}{c}$, $\underset{\cdot\cdot}{c}$: and conſequently all the Values of the reſt of the Increments of x, viz. $\underset{::}{x}$, $\underset{\because}{x}$, &c. will be expreſs'd by the ſame Symbols. Whence it follows that to determine the Values of the Symbols c, $\underset{\cdot}{c}$, $\underset{\cdot\cdot}{c}$, there may be taken three Conditions relating to the Values of x, $\underset{\cdot}{x}$, $\underset{\cdot\cdot}{x}$, $\underset{\therefore}{x}$, $\underset{::}{x}$, &c. promiſcuouſly, as the fourth Propoſition directs. The ſame Rule holds good in the Method of Fluxions. For Example, having given the Æquation $a z^2 + x^2 \ddot{x} = 0$, if it be propoſed to deſcribe the Curve that it belongs to; by the fourth Propoſition it may alſo be required as a Condition, that the Curve ſhall paſs through two given Points; that it ſhall touch two given Lines; that it ſhall paſs thro' a given Point, and when it cuts a given Line ſhall have a given Degree of Curvature; or that it ſhall have any other two Circumſtances that depend upon the Values of the third, fourth or other Fluxions. Theſe Conditions that attend Incremental or Fluxional Equations, I don't know to have been ſufficiently taken notice of by any Body: but they ought well to be attended to in the Inverſe Methods; the Solutions of particular Problems being never perfect, unleſs there be proviſion made for the ſatisfying of them, by the indetermined Coefficients in the Equation that contains the Solution of the Problem. Examples of this may be ſeen in Prop. 17 and 18, where I give the Solution of the Problems concerning the *Iſoperimeter*, and the *Catenaria*.

The ſixth Propoſition contains the general Explanation of the Inverſe Method both of Fluxions and of Increments, which conſiſts in the Solution of this Problem. Having given the Relations of the Increments, or of the Fluxions of ſeveral Quantities, whether they be conſidered

red with their proper Integrals or with their proper Fluents or not; to find the Relations of the Integrals or of the Fluents, freed from their Increments or from their Fluxions. The Direction I have given for finding the Solution in finite Terms is but tentative. And I must confess I know of no other Method that is general for all Cases. For I can find no certain Rule to judge in general, whether any proposed Equation, involving Increments or involving Fluxions, can be resolved in finite Terms. For this Reason we are obliged to seek the general Solution in infinite Serieses; which when they break off, or when they can any way be reduced to finite Terms, they then contain the Solutions which we always hope for. The Method of finding these Serieses is explain'd in the eighth Proposition, and that is by means of a Series that is demonstrated in the seventh Proposition. And this I take to be the only genuine and general Solution of the inverse Methods. For in this Solution you always have those indetermined Coefficients, which are necessary to adapt the Equation that is found to the Conditions of the Problem proposed. For want of this Circumstance all other Methods are imperfect; and particularly Sir *Isaac Newton*'s Method of finding Serieses by a Rular and Parallelograms labours under this Difficulty, because it brings no new Coefficients into the resulting Equation, which may afterwards be determined by the Conditions of the Problem. However because this Method is very ingenious and very elegant, I thought it proper to explain it in the following (*viz.* the *9th*) Prop. The *10th*, *11th*, and *12th* Propositions conclude the first Part, and in them I treat of the manner of finding the Integral or the Fluent, having given the Expression of a particular Increment, or of a particular Fluxion of it; without being involved with the Integrals, or with the Fluents, or with any other Increments, or with any other Fluxions of it. This is a particular Caſe of the Inverſe Method, but for its great uſefulneſs I thought it deſerved particularly to be taken notice of. This Problem is treated of in general in the 10*th* Propoſition. The Method of ſolving it in finite Terms is only tentative; and when that does not ſucceed, recourſe muſt neceſſarily be had to the Solution by a Series in the 8*th* Propoſition. In the 11*th* and 12*th* Propoſitions I have ſhew'd how Serieſes may be conveniently found, in ſome particular Caſes when Fluxions are propoſed.

In the ſecond Part I have endeavour'd to ſhew the Uſefulneſs of theſe Methods in the Solution of ſeveral Problems; The 13*th* Propoſition is much the ſame with Sir *Iſaac Newton*'s *Methodus Differentialis*, when the Ordinates are at equal Diſtances: and in an Example at the End of this Propoſition I have ſhew'd how eaſily Sir *Iſaac Newton*'s Series for expreſſing the Dignity of a Binomial may be found by this Incremental Method. The 14*th* Propoſition ſhews in ſome meaſure how this Method may be of uſe in ſumming up of Arithmetical Serieſes. In the 15*th* Propoſition I ſhew by ſome Examples how the Proportions of the Fluxions are to be found in Geometrical Figures; from whence immediately flows the Method of finding the Radiuſes of their inoſculating Circles, the Invention of the Points of contrary Flexure, and the Solution of other Problems of the like nature. In the 16*th* Propoſition I ſhew how the Method of Fluxions is to be applied to the Quadrature of all ſorts of Curves. In the following Propoſition I give a general Solution of the Problem of the *Iſoperimeter*, which has been treated of by the two famous Mathematical Brothers the *Bernulli*'s. In the 18*th* Propoſition I give the Solution of the Problem about the *Catenaria*, not only when the Chain is of a given Thickneſs every where, but in general, when its Thickneſs alters according to any given Law. In the

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following Propoſition I ſhew the *Fornix* or Arch which ſupports its own Weight to be the ſame with the *Catenaria*. In the two next Propoſitions I ſhew how to find the Figures of pliable Surfaces which are charged with the Weight of a Fluid. In the 22*d* and 23*d* Propoſitions I treat of the Motion of a Muſical String, and give the Solution of this Problem: To find the Number of Vibrations that a String will make in a certain time, having given its Length, its Weight, and the Weight that ſtretches it. This Problem I take to be entirely new, and in the Solution of it (in the laſt part of Prop. 23.) there is a remarkable Inſtance of the Uſefulneſs of the Method of firſt and laſt Ratio's. The 24*th* Propoſition gives the Invention of the Center of Oſcillation of all Bodies; and in the 25*th* Propoſition I have given the Inveſtigation of the Center of Percuſſion. It is known that this Problem is ſolved by the ſame *Calculus* as the foregoing; wherefore it is generally thought that theſe two Centers are the ſame. But that is a Miſtake, becauſe the Center of Oſcillation can be but one Point; but the Center of Percuſſion may be any where in a certain Line, which this Propoſition ſhews how to find. There is an Error in this Propoſition, which I was not ſenſible of till after the Book was publiſh'd, wherefore I take this Opportunity of correcting of it. It does not affect the Reaſoning by which I find the Diſtance of the Center of Percuſſion from the Axis of Rotation; but it is this, that I ſuppoſed the Center of Percuſſion to be in the Plane paſſing thro' the Center of Gravity, and perpendicular to the Axis of Rotation: which is a Miſtake. It is corrected by the following Propoſition.

PROP.

PROP. PROB.

To find the Distance of the Center of Percussion from the Plane passing thro' the Center of Gravity and perpendicular to the Axis of Rotation.

SOLUTION.

Let the sixth Figure be suppos'd in the Plane passing thro' the Axis of Rotation, and in which the Center of Percussion is sought.

Let *A B* be the Axis of Rotation, *A G C* be the Intersection of this Figure with the Plane passing thro' the Center of Gravity, and perpendicular to the Axis of Rotation, *G* be the Point whereon a Line, rais'd perpendicular to this Figure, will pass thro' the Center of Gravity; *B E* be a Line parallel to *A G* wherein is the Center of *Percussion*. Then to find the Distance *A B*, let *p* stand for an Element of the Body propos'd standing perpendicularly on any point *D* Draw *D C* perpendicular to *A G C*. and *A B* will be equal to the Summ of all the Quantities $p \times GC \times CD$ taken with their proper Signs, divided by the Body it self multiplied into the Distance *A G*.

Having thus found the Distance *A B*, suppose the Plane of the Figure in *Prop.* 25. to cut the present Figure at right Angles in the Line *B E*, and the Center of Percussion will be rightly determined by that Proposition.

The 26*th* Proposition shews how to determine the Density of the Air at any Distance from the Center of the Earth supposing the Density always to be proportional to the compressing Force, and that the Power of Gravitation is reciprocally as the Distances from the Center of the Earth.

The laſt Propoſition ſhews how to find the Refraction of a Ray of Light in its paſſage thro' the Atmoſphere, upon the Suppoſition that Light is a Body, and that the Refraction of it is cauſed by the Attraction of the Bodies the Rays approach to. In this Propoſition there is a remarkable Inſtance of the Uſefulneſs of the Method of Increments in finding the Coefficients of a Series, which according to the Values of a certain Symbol, as *n*, expreſſes both all the Fluents, and all the Fluxions of a certain Quantity.

II. Ludovici Ferdinandi Marſilii *Diſſertatio de* Generatione Fungorum. *Rom.* 1714. 4to.

THis Author tells us that he gave his youthful Inclinations to the Study of the Mathematicks and Obſervations of Nature, under the Tuition of the celebrated *Malpigius*, and *Lelius Triumfettus* Botanick Profeſſor of *Bononia* : and amongſt the various Productions of Nature, his chief Delight was in the Contemplation of the ſudden Growth and various ſorts of *Muſhrooms* which both the Earth and Trees brought forth. Of the firſt Kind he obſerved the greateſt Number to ariſe in Camps, produced from the Horſes Dung, and are commonly called *Prataiouli*.

In the Years 1699 and 1700, being then in *Croatia* and *Tranſylvania*, in the Armies there, he made a large Volume of Deſigns of *Fungi*, which he ſent to *Triumfetti* to put in order, who added a great Number to them of ſuch as he found about *Bononia* ; yet after all the moſt diligent Search, he could never find them to produce any Seed either in their Gills or other Parts.

The Origin and Generation of *Muſhrooms* he ſays is not eaſy to demonſtrate, ſince both the Antients and Moderns disagree very much about it. The late Botanists seem to be of three different Sentiments concerning their Produce. Mr. *Ray*, Dr. *Sherrard*, Mr. *Doody*, *Boccone* and *Mentzelius* having observed some *Mushrooms* to have had Seed, were of opinion others might have the same Original. *Clusius* and *John Baptista Porta* had in some also observed their Seed: Others, *viz.* *Sharrock* and the accurate *Malpigius*, who could not find any Seed in them, altho' with the Assistance of Microscopes, did suppose they might be produced by Pieces of themselves, carried by the Winds from place to place, as other Plants are by Slips and Offsetts.

The third Opinion, which he says most agree in, is that they arise from Putrefaction, or a Mixture of certain Salts, Sulphur and Earth impregnated with the Dung of Beasts.

The *Fungus seminifer Campaniformis Mentzelii*, &c. being the Mushroom which first gave the occasion of the Opinion of their having Seed, this nice Author has accurately figured and observed, and supposes with others that these seedlike Bodies may be the *Ovaria* of some Insects; and the rather because they are so very large in proportion to the smallness of the *Mushroom*: and that they had often been sowed by Dr. *Amadoes* a curious Botanist, without any Success towards raising them. From whence he concludes these Bodies ought to have another Denomination than Seed; neither is he of the Opinion that they are produced by parts of themselves.

In his Division of *Mushrooms* he first treats of the *Truffles* and their Increase, Situation and Soyl, Colour, Tast and Consistence. He next proceeds to soft Mushrooms, such as he observed in his own Garden; which having in the Spring been meliorated with Horse-dung, about the middle of *June* there sprung up divers of that sort which the *Italians* call *Prataiuoli*, amongst a Bed of *Lettice*. These continued till near the midst of *August* before they went

 off.

off. Of these and some other Kinds he accurately figures the first Shootings and Fibres.

His next Tribe are such as grow from Wood, but yet are themselves soft. Of these he observes three Kinds; the first a large one in his Window, out of a piece of Firwood which it had often rained on; with two smaller sorts from some rotten Boards in his Garden. All these he figures both in their natural and divided States, as also Microscopically

Treating of hard woody *Mushrooms* (of which he also gives you some accurate Figures) he observes they rarely appear on the Trees, in *Germany* and *Croatia*, before they are twenty or thirty Years old; but most commonly when forty or fifty: and the Original of them he attributes chiefly to the Rottenness of the Wood, and says they generally break out in the Spring, when the Leaves begin to shoot. And that usually they grow below the middle of the Trees, and are cause of so much Decay in them, that they often die in three or four Years.

It may not here be amiss to subjoin what Dr. *Lancisius* communicates to our Author, concerning the *Lapis Fungarius, viz.* that altho' this *Mushroom-producer* has the Name of a Stone, it ought not to be reckoned of that *Genus*, it being really no other than a Mass or *Congeries* of Roots, Seeds and Juices coagulated with Earth into, as it were, a stony Substance. Upon which pouring Water and setting it in a warm Place, it loosens its hardned Substance; and by mollifying its Fibres and moistning its concrete Juices, out of the Cliffs and Chinks thereof the *Mushrooms* spring, as they do in other places from simple Dung and loose Earth And it is also farther to be noted, that when this stony Mass has thus yielded these its Offspring, the Remains grows light, porous and decay'd, its nutritive Juices being then exhausted.

FINIS.